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ADDITIONAL DEGREES OF FREEDOM IN SKYRMION MOTION*

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Abstract

We consider quantization of chiral solitons with baryon number $B > 1$. Classical solitons are obtained within a framework of the variational approach. From the form of the soliton solution it can be seen that besides the group of symmetry describing transformations of the configuration as whole there are additional symmetries corresponding to internal transformations. Taking into account the additional degrees of freedom leads to some sort of spin alignment for light nuclei and gives constraints on their spectra.

I. INTRODUCTION

Considerable recent interest in the Skyrme model [1] as a possible theory of strongly interacting particles is a consequence of the hope that meson effective Lagrangians can bridge the gulf between quantum chromodynamics (QCD) and the known theory of nuclear structure.

Although everyone believes that physics of any nucleus is also described by the QCD Lagrangian, no one has been able to obtain the basic properties of nuclei in terms of quark and gluon fields. It is very difficult to analyze the dynamics of the quark and gluon fields in low-energy quantum chromodynamics because of the large coupling constant.

Searching for a small parameter in QCD, 't Hooft proposed the idea of considering QCD with a large (tending to infinity) number of colors N_c. Later, Witten showed that if the limit $N_C \rightarrow \infty$ exists, then QCD is a theory of effective meson fields with local interactions with a coupling constants of an order of $1/N_c$. Moreover, in this limit the baryon masses prove to be of an order of N_c, while the number of colors completely drops out of the equations determining the size and structure of the baryons.

It is well known that nonlinear theories can have solutions corresponding to localized objects of finite size — solitons [2]– with the analogous dependence of the size on the coupling constant. Therefore, Witten's result leads to the description of baryons as solitons of an effective meson theory. This picture does not require any further reference to the quark origin of the effective Lagrangian. A theory of just this type was proposed by Skyrme in 1961-1962 [1].

Nonlinear chiral theories naturally lead to soliton sectors. Already at the classical level, chiral solitons are very similar to hadrons. They carry a definite, rigorously conserved topological charge. This localized charge is a good candidate for the baryon number. Chiral solitons are extended, strongly interacting objects. They have very large mass compared with the masses of the fields involved in the Lagrangian.

These features plus rich spectrum of generated states make chiral dynamics a very attractive theory for low-energy phenomena in strong interaction physics.

Restricting ourselves to the simplest model of this type - the Skyrme model, we probably cannot hope for good quantitative agreement with the experimental data, but we can obtain a qualitatively good description of the fundamental regularities characterizing a system of strongly interacting particles which would support the idea that baryons are solitons of the effective meson Lagrangian.

The Skyrme model gives us a straightforward way for constructing a system with an arbitrary baryon charge. We have to look for solitons of classical fields with corresponding topological charge and then to quantize solitonic degrees of freedom to obtain an object with nuclear quantum numbers.

Recently a specific variational ansatz was proposed independently in [3] and [4]. This ansatz obeys the symmetry conditions formulated in [6], [7] and, being very simple, gives the possibility to do one more step in analytical analysis of the problem and to take into account vibrational modes, for example, the monopole one, in a simple way. This analyses gives a natural explanation of the origin of the ansatz used earlier in [8] and also gives some new solutions.

To obtain quantum spectra of multibaryon, one has to perform quantization of pion field around the multisoliton classic field configuration. It is well known that the Lagrangian describing the quantum pion field contains zero modes, which are determined by the symmetry group of the classical soliton solution. The zero mode should be treated in a special way. The most convenient method is to introduce corresponding collective coordinates. This leads to the interpretation of the soliton as a quantum particle moving in the collective coordinate space. As will be shown, the multisoliton solutions obtained with the anzatz [3,4] possess additional internal group of symmetry. As a consequence, new restrictions on the spectra of multibaryons arise.

II. ANSATZ AND SOLUTIONS FOR STATIC EQUATIONS

Here we follow our paper [9] (see also [5]). In a variational form of the chiral field U

$$U(\vec{r}) = \cos F(r) + i(\vec{\tau} \cdot \vec{N}) \sin F(r). \tag{1}$$

we use the following assumption about the configuration of the isotopic vector field $\vec{N}$:

$$\vec{N} = \{\cos(\Phi(\phi, \theta)) \cdot \sin(T(\theta)), \ \sin(\Phi(\phi, \theta)) \cdot \sin(T(\theta)), \ \cos(T(\theta))\}. \tag{2}$$

In Eq.(2) $\Phi(\phi)$, $T(\theta)$ are some arbitrary functions of angles (θ, ϕ) of the vector $\vec{r}$ in the spherical coordinate system.

Let us consider the Lagrangian density $\mathcal{L}$ for the stationary solution:

$$\mathcal{L} = \frac{F_\pi^2}{16} \cdot Tr(L_k L_k) + \frac{1}{32e^2} \cdot Tr\left[L_k, L_i\right]^2. \tag{3}$$

Here $L_k = U^+ \partial_k U$ are the left currents.

Variation of the functional $L = \int \mathcal{L} d\vec{r}$ with respect to Φ leads to an equation which has a solution of the type

$$\Phi(\theta, \phi) = k(\theta) \cdot \phi + c(\theta)$$

with a constraint

$$\frac{\partial}{\partial \theta} \left[\sin^2 T(\theta) \cdot \sin \theta \cdot \frac{\partial \Phi(\theta, \phi)}{\partial \theta} \right] = 0. \tag{4}$$

It is easily seen from eq. (4) (see also [9]) that functions $k(\theta)$ and $c(\theta)$ may be piecewise constant functions (step functions):

$$\Phi(\theta, \phi) = \begin{cases} k^{(1)} \cdot \phi + \phi_0^{(1)} , & \text{for } 0 \leq \theta < \theta_1 , \\[2mm] k^{(2)} \cdot \phi + \phi_0^{(2)} , & \text{for } \theta_1 \leq \theta < \theta_2 , \\[2mm] \quad \cdot \quad \cdot \quad \cdot \\[2mm] k^{(n)} \cdot \phi + \phi_0^{(n)} , & \text{for } \theta_{n-1} \leq \theta < \pi . \end{cases}$$

Moreover, $k^{(m)}$ must be integer in any region $\theta_m \leq \theta \leq \theta_{m+1}$, where θ_m, θ_{m+1} are successive points of discontinuity of $\partial \Phi_i(\theta, \phi)/\partial \theta$. The positions of these are the points determined by the condition

$$T(\theta_m) = m \cdot \pi , \quad T(\pi) = n \cdot \pi \tag{5}$$

with integer m, as follows from eq.(4).

Now we have the following expression for the mass of a soliton

$$M = \gamma \cdot \sum_{m=1}^{n} \left[a^{(m)} \cdot A^{(m)} + b^{(m)} \cdot B^{(m)} + C^{(m)} \right], \tag{6}$$

where $\gamma = \pi \cdot F_\pi / e$ and $x = F_\pi \cdot e \cdot r$ and the $a^{(m)}$, $b^{(m)}$ and $A^{(m)}$, $B^{(m)}$, $C^{(m)}$ are some integrals of T, T', F, F' on θ and x. The functions $F^{(m)}(x)$ and $T^{(m)}(\theta)$ have to obey the equations [9] in arbitrary space region with given number $k^{(m)}$.

III. THE NUMBER OF ZERO MODES

Let us consider the quantization of the static multibaryon configuration (2),(4). This procedure implies that the pion field is represented in the form of a superposition of the background classical field plus small (quantum) fluctuations around it:

$$\phi(\vec{x}, t) = \Phi_c(\vec{x}) + \phi_q(\vec{x}, t). \tag{7}$$

Then action for quantum pion field can be expanded into series in ϕ_q:

$$S(\phi) = S_0(\Phi_c) + \int dx\, dy\, \phi^a(x) \cdot \left(\left. \frac{\delta^2 S(\phi)}{\delta \phi^a(x) \delta \phi^b(y)} \right|_{\phi = \Phi_c} \right) \cdot \phi^b(y) + \ldots, \tag{8}$$

linear term vanishes as a consequence of equations of motion.

The well-known problem arises due to the zero modes in ((8)). In the terms of the path integral quantization it means that some of the integrations in the functional space are non-Gaussian and should be carried out exactly (rather than in the saddle – point approximation).

First question is about the number of zero modes. The reason for these zero modes is that a soliton solution breaks explicitly some of the symmetries of the initial Lagrangian, and each mode restores relevant symmetry of the partition function. So, usual way to treat them is to extract the volume of the symmetry group, in particular, introducing a set of time– dependent collective coordinates $\alpha(t)$. Thus the measure in the path integral can be modified by inserting the Faddeev – Popov unity into the form

$$Z = \int D\pi e^{iS(\Phi_c;\pi)} = \int D\{\alpha\} \int D\pi' e^{iS(\Phi_c, \{\alpha\};\pi)}, \tag{9}$$

where prime denotes that zero modes are excluded from the path integral measure over the pion field.

The collective coordinates can be chosen as parameters of the soliton solution $\Phi_c(\vec{x}; t) = \Phi_c(\vec{x}; \{\alpha(t)\})$, the classical action $S_0(\Phi; \{\alpha\})$ being in fact independent on α's. First of all, the parameters are the ones defining the global transformations of a soliton — the coordinate of the center X, and matrices of the orientation in configurational and iso– spaces I, R:

$$U(\vec{x}; t) = e^{i\vec{p}\vec{X}} \cdot e^{iTI} \cdot e^{-iSR} \cdot U_0(\vec{x}) = \exp\left\{ i\tau^i I^{ij}(t) N^j \left(R_{kl}^{(-1)}(t)(x_k - X_k(t)) \right) \right\} \tag{10}$$

Here we denote generators of the translation and rotations in space and iso– space as P, M and T respectively.

In general, the multisoliton field configuration ((2),(4)) allows for wider group of symmetry due to specific form of the ansatz. The action can be seen to be independent on the parameters $\varphi_0^{(i)}$, which define the orientation (in the xy– plane) of each sector $\theta \in [\theta^{(i-1)}, \theta^{(i)}]$.

However, not all of the parameters we have introduced are in fact independent. To see this, let us represent matrix I and (R) (and the relevant generators) as a composition of the two parts:

$$I = I_\perp \cdot I_3, \quad R = R_\perp \cdot R_3 \tag{11}$$

where R_3 (I_3) describes the rotation around the z– (third) axis in space (isospace), and $R_\perp$ $(I_\perp)$ describes the rotation around an axis lying in the xy– (12–) plane. Then, note that instead of the parameters $\varphi_0^{(i)}$ the set of matrices $R^{(i)}(\theta)$ may be introduced, so that

$$R^{(i)}(\theta) = \begin{cases} R_3(\phi_0^{(i)}), & \theta \in [\theta^{(i)}, \theta^{(i+1)}], \\ 1, & \text{otherwise.} \end{cases} \tag{12}$$

Of course, $R^{(i)} \cdot R^{(j)} = R^{(j)} \cdot R^{(i)}$. Let us define, in the analogy with (12), the set of operators $S^{(i)}(\theta)$ and $T_3^{(i)}(\theta)$, which rotate the i-th sectors around the z– (third) axis independently. It is easy to check that

$$S^{(i)} - k^{(i)} \cdot T_3^{(i)} = 0, \tag{13}$$

and

$$M_3 = \sum_{i=1}^n S^{(i)} = \sum_{i=1}^n k^{(i)} \cdot T_3^{(i)}, \quad T_3 = \sum_{i=1}^n T_3^{(i)}. \tag{14}$$

As a result, we see that independent operators of the space and iso– space rotations may be chosen as $S_\perp$, $T_\perp$ and the set of $T_3^{(i)}$ (or one can equivalently choose another operator basis of the same dimension). So, independent collective coordinates are $R_\perp$, $I_\perp$ and the set of $I^{(i)}$

IV. LAGRANGIAN IN THE COLLECTIVE COORDINATE VARIABLES

We want to find the spectra of low – laying quantum states of the multi-baryons which correspond to the classical multisoliton field configuration (2,(4)). One performs this most straightforward by means of the canonical quantization method.

For our purpose the zero modes corresponding to the rotational symmetries seem most interesting, since they determine the rotational spectrum structure of low – laying multibaryon states. Therefore, we restrict ourselves here to considering the zero modes only.

A natural way to proceed is to rewrite the Lagrangian in terms of the independent collective coordinates and their time derivatives (velocities) and to derive the Hamiltonian. However, it is more instructive to keep the overfull set of the parameters: R, I and set of $\varphi^{(i)}$, i.e. not to separate out the overall rotation and iso– rotation around the z– (third) axis. We will obtain the constraints (13) again at the end of the calculations.

It is convenient to define the angular velocities by

$$R_{ik}^{-1} \dot{R}_{kj} = \epsilon_{ijl} \Omega_l, \quad \dot{I}_{kj} I_{ik}^{-1} = \epsilon_{ijl} \Omega_l. \tag{15}$$

Inserting Eqs. (10), (11) and (12) into the Lagrangian, we obtain

$$L = -M + \frac{F_\pi^2}{16} \cdot \int \text{Tr}(L_0 L_0) d^3x + \frac{1}{16e^2} \cdot \int \text{Tr}[L_0, L_i]^2 d^3x = -M + L' \tag{16}$$

and

$$L' = \frac{1}{2} \left\{ \vec{\Omega}_\perp^2 Q_S + \vec{\omega}_\perp^2 Q_T + \sum_{i=1}^N \left(\omega_3 k^{(i)} + \dot{\phi}^{(i)} + \Omega_3 \right)^2 C^{(i)} + 2(\Omega_1 \omega_1 K_1 + \Omega_2 \omega_2 K_2) \right\}. \tag{17}$$

Here $\vec{\Omega}_\perp^2 = \Omega_1^2 + \Omega_2^2$, $\vec{\omega}_\perp^2 = \omega_1^2 + \omega_2^2$ and $\vec{\omega}_\perp \cdot \vec{\Omega}_\perp = \omega_1 \Omega_1 + \omega_2 \Omega_2$, M is the classical energy of a soliton.

$K_{1,2}$ do not vanish if there is at least one sector with $|k| = 1$. We will consider multisolitons with all $k^{(i)}$ positive. In this case $K_1 = K_2 = K$ and the sum in the last parenthesis gives $K\vec{\Omega}_\perp \cdot \vec{\omega}_\perp$, so the system is a symmetrical rotator.

The quantities Q's and K's in the above equation may be considered as a sum of the independent contributions from different sectors $\theta \in [\theta_{i-1}, \theta_i]$:

$$Q_{S,T} = \sum_{i=1}^N Q_{S,T}^{(i)}, \quad K_{1,2} = \sum_{i=1}^N K_{1,2}^{(i)}. \tag{18}$$

Explicit expressions for all the parameters in eq. (17) are given in Appendix A.

V. THE HAMILTONIAN FOR A QUANTIZED MULTIBARYON

Let us introduce the canonical momenta conjugated to each of the collective coordinates as follows

$$T_m = \frac{\delta L}{\delta \omega_m}, \quad S_m = \frac{\delta L}{\delta \Omega_m}, \quad W^{(i)} = \frac{\delta L}{\delta \varphi_0^{(i)}}. \tag{19}$$

After the canonical transformation one arrives at the expression for the Hamiltonian

$$H = M + \frac{\vec{S}^2}{2Q'_S} + \frac{\vec{T}^2}{2Q'_T} - \frac{S_3^2}{2Q'_S} - \frac{T_3^2}{2Q'_T} + \frac{\vec{S}\cdot\vec{T}}{Q_{ST}} + \sum_{i=1}^{N} \frac{W^{(i)2}}{2C^{(i)}}, \tag{20}$$

where T_3 and S_3 are not independent and are related to the set of $W^{(i)}$ via the constraints

$$T_3 = \sum_{i=1}^{N} W^{(i)}, \qquad T_3^{(i)} = W^{(i)}$$
$$S_3 = \sum_{i=1}^{N} k^{(i)} \cdot W^{(i)}, \quad S^{(i)} = k^{(i)} \cdot W^{(i)}, \tag{21}$$

which are consistent with Eqs. (13) and (14).

Note that these relations hold only in the internal frame.

The new parameters Q'_S and Q'_T in Eq.20 coincide with Q_S and Q_T respectively and $Q_{ST} \to \infty$, if no sectors with $|k^{(i)}| = 1$ are presented. Otherwise,

$$Q'_S = Q_S - \frac{K^2}{Q_T}, \quad Q'_T = Q_T - \frac{K^2}{Q_S}, \quad Q_{ST} = -K + \frac{Q_s Q_T}{K}. \tag{22}$$

VI. QUANTUM SPECTRA OF MULTISOLITONS AND NUMERICAL RESULTS

We want to construct quantum states of a multisoliton as compositions of quantum states of individual sectors (regions) $[\theta_{(i-1)}, \theta_{(i)}]$, which have definite spin and isospin quantum numbers:

$$|S^{(i)}, T^{(i)}, S_3^{(i)} = k^{(i)} T_3^{(i)}\rangle. \tag{23}$$

To this end, we define the most general composition and then step by step apply the restrictions which follow from the form of the Hamiltonian and from the rotational symmetry.

Note, that this problem is different from the standard problem of constructing quantum states of a system of spinning particles. It is due to the specific form of our Hamiltonian, which possesses definite quantum numbers not only for total spin and isospin with their third components but also for the operators $T_3^{(i)}$ (and, as a consequence, $S^{(}(i)_3 = k^{(i)} \cdot T_3^{(i)}$) for each of the sectors.

Global spin and isospin rotational symmetry dictates that a multisoliton quantum state should have the form of a linear combination

$$\Psi(S, T, S_3, T_3, T_3^{(i)}) = \sum_{T^{(i)}, T_3^{(i)'}} c_{T^{(i)}, T_3^{(i)'}} \cdot \psi(S, T, S_3, T_3, T^{(i)}, T_3^{(i)'}) \tag{24}$$

of the expressions

$$\psi(S, T, S_3, T_3, T^{(i)}, T_3^{(i)}) = \sum_{T_3^{(i)}=-T^{(i)}}^{T^{(i)}} C_{\{S_3^{(i)}\}}^{S,S_3} C_{\{T_3^{(i)}\}}^{T,T_3} \prod |S^{(i)}, T^{(i)}, S_3^{(i)} = k^{(i)} T_3^{(i)}\rangle. \tag{25}$$

Here C are the $3nJ$ –symbols, and we have put the relation (21).

For the sake of simplicity of our consideration, we will proceed with the case of the multisoliton configuration with two sectors and dismiss the spin quantum numbers S, S_3; it easily can be extended to a multisoliton with arbitrary number of sectors and for the full set of the variables.

First of all, from the requirement that the multisoliton state must be an eigenstate of the operators $\hat{T}_3, \hat{T}_3^{(1)}, \hat{T}_3^{(2)}$ we see that the sum (24,25) contains only one term with

$$T_3 = T_3^{(1)} + T_3^{(2)}, T^{(i)} = |T_3^{(i)}|. \tag{26}$$

Furthermore, since $\hat{T} = \hat{T}^{(1)} + \hat{T}^{(2)}$, for the operator of the total isospin squared we have:

$$\hat{T}^2 = \hat{T^{(1)}}^2 + \hat{T^{(2)}}^2 + 2 \cdot \hat{T}_3^{(1)} \cdot \hat{T}_3^{(2)} + \hat{T}_+^{(1)} \cdot \hat{T}_-^{(2)} + \hat{T}_-^{(1)} \cdot \hat{T}_+^{(2)} \tag{27}$$

On the other hand,

$$\hat{T}^2 |T, T_3; T_3^{(1)}, T_3^{(2)}\rangle = T(T+1)|T, T_3; T_3^{(1)}, T_3^{(2)}\rangle \tag{28}$$

and $T = T^{(1)} + T^{(2)}$, which is consistent with (26) and (27) only if two last terms in (27) vanish. It leads to the conclusion that both $T_3^{(1)}, T_3^{(2)}$ have the same sign.

As the result, we see that the multisoliton quantum state $|S, T, S_3, T_3; T^{(i)}\rangle$ has the form of a product of the sector's quantum states (23) with the relations

$$T = \sum T^{(i)}, \quad S = \sum S^{(i)}, \quad T_3 = \sum T_3^{(i)}, \quad S_3 = \sum S_3^{(i)}, \tag{29}$$

$$T_3^{(i)} = +T^{(i)} \ (or \ T_3^{(i)} = -T^{(i)}), \tag{30}$$

$$S_3^{(i)} = k^{(i)} \cdot T_3^{(i)}, \quad S^{(i)} = |S_3^{(i)}|. \tag{31}$$

Substituting these into the Hamiltonian (20) gives the energy of a soliton:

$$E = M + \frac{S}{2Q_S} + \frac{T}{2Q_T} + \frac{ST}{Q_{ST}} + \sum_j \frac{(T_3^{(j)})^2}{2C_j} \tag{32}$$

and the constraint on its spin and isospin quantum numbers

$$T = max \left| \sum_j T_3^{(j)} \right|. \tag{33}$$

The corresponding calculations of the rotational energies for the solitons with baryon number three have been worked out. In [9] it was shown that the toroidal configuration ($L = 1$, $k = \{3\}$) can not have t and ^{3}He quantum numbers. In fact, the corresponding quantum numbers are $T = 1/2$, $S = 3/2$ but not $T = 1/2$, $S = 1/2$ which have to be for t and ^{3}He. It is easy to see from the last formulas that only non-toroidal configuration ($L = 2$, $k = \{1,2\}$) can have correct quantum numbers after quantization. Their masses are equal to each other (a possible coulomb mass differences are not considered). From equation (32) one obtains that the rotational motion energy is about $23.5 \ MeV$. Classical part of the mass M in eq.(32) is $2987 \ MeV$. The values of the constants $F_\pi = 109.45 \ MeV$ and $e = 4.138$, which have been used in our calculations, correspond to the values at which the smallest masses of the solitons with $B = 4$ and $B = 12$ coincide with the masses of the ^{4}He and ^{12}C nuclei [10]. It is evident that the adiabatic rotation motion approximation is more convenient for nuclei than for nucleon.

VII. CONCLUSION

The quantization procedure including the additional new zero modes for the non-toroidal soliton configurations has been developed. Obtained effective Hamiltonian leads to new formulas for eigenvalue spectra of the quantum solitons due to the additional constraints we have obtained for the quantum numbers of considered solitons. The non-toroidal solitons ($L = 2$, $k = \{1,2\}$) have correct quantum numbers of t and ^{3}He after quantization in contrast to the pure toroidal configurations. We have to note here that it is only taking into account the additional zero modes that leads to this successful picture.

VIII. ACKNOWLEDGMENTS

APPENDIX A: FORMULAS FOR THE MOMENTS OF INERTIA

Here we list the explicit expressions for the parameters in eq.17. It is customary to use the dimensionless variable $x = \frac{e}{F_\pi} r$ instead of r.

$$Q_T^{(i)} = \frac{\pi}{F_\pi e^3} \int_{\theta_{i-1}}^{\theta_i} \sin\theta d\theta \int_0^\infty x^2 dx \left\{ -\frac{\sin^4 F}{x^2} \left(k_i^2 \frac{\sin^2 T}{\sin^2\theta} \cos^2 T + (T')^2 \right) \right.$$
$$\left. + \sin^2 F \cdot \left[\frac{1}{4} + (F')^2 + \left(\frac{k_i^2 \sin^2 T}{\sin^2\theta} + (T')^2 \right) \frac{\sin^2 F}{x^2} \right] \cdot (1 + \cos^2 T) \right\}, \tag{A1}$$

$$Q_S^{(i)} = \frac{\pi}{F_\pi e^3} \int_{\theta_{i-1}}^{\theta_i} \sin\theta d\theta \int_0^\infty x^2 dx \left\{ -\frac{\sin^4 F}{x^2} \left(k_i^4 \frac{\sin^4 T}{\sin^4\theta} \cos^2 T + (T')^4 \right) \right.$$
$$+ \sin^2 F \cdot \left[\frac{1}{4} + (F')^2 + \left(\frac{k_i^2 \sin^2 T}{\sin^2\theta} + (T')^2 \right) \frac{\sin^2 F}{x^2} \right] \tag{A2}$$
$$\left. \cdot \left(k_i^2 \frac{\sin^2 T}{\sin^2\theta} \cos^2\theta + (T')^2 \right) \right\},$$

$$K_1^{(i)} = \delta_{k_i,1} \frac{\pi}{F_\pi e^3} \int_{\theta_{i-1}}^{\theta_i} \sin\theta d\theta \int_0^\infty x^2 dx \left\{ -\frac{\sin^4 F}{x^2} \right.$$
$$\left. + \sin^2 F \cdot \left[\frac{1}{4} + (F')^2 + \left(\frac{k_i^2 \sin^2 T}{\sin^2\theta} + (T')^2 \right) \frac{\sin^2 F}{x^2} \right] \right\} \tag{A3}$$
$$\cdot \left(T' \sin\theta - \sin T \cos T \frac{\cos\theta}{\sin\theta} \right),$$

$$K_2^{(i)} = k_i \cdot K_1^{(i)}.$$

$$C_i = \frac{\pi}{F_\pi e^3} \int\limits_{\theta_{i-1}}^{\theta_i} \sin\theta d\theta \int\limits_0^\infty x^2 dx \left\{ -\frac{\sin^4 F}{x^2} k_i^2 \sin^2 T \right.$$
$$\left. + \sin^2 F \cdot \left[\frac{1}{4} + (F')^2 + \left(\frac{k_i^2 \sin^2 T}{\sin^2\theta} + (T')^2 \right) \frac{\sin^2 F}{x^2} \right] \right\},$$

(A4)

REFERENCES

[1] *Skyrme T.H.R.*: Nucl.Phys. **31**, 556, (1962).

[2] *Faddeev L.D. and Korepin V.E.*: Phys.Rep. **42C**, 1, (1978).;
Rajaraman R. in: Solitons and instantons, North-Holland, Amsterdam (1982).

[3] *Nikolaev V.A., Tkachev O.G.*: JINR Preprint E4-89-56, Dubna, (1989).;
Nikolaev V.A., Tkachev O.G.: TRIUMF (FEW BODY XII). TRI-89-2, Vancouver, F25, (1989).

[4] Sorace E., Tarlini M. Phys.Lett. B232, 154 (1989).

[5] *Nikolaeva R.M., Nikolaev V.A., Tkachev O.G.*: Jour. of Nucl. Phys. **56(7)**, 173, (1993). *Nikolaeva R.M., Nikolaev V.A., Tkachev O.G.*: J.Phys.G: Nucl.Phys. **18**, 1149, (1992).

[6] *Verbaarshot J.J.M.*: Phys.Lett. **B195**, 235, (1987).

[7] *Manton N.S.*: Phys.Lett. **B192**, 177, (1987).

[8] *Weigel H., Schwesinger B., Holzwarth G.*: Phys.Lett. **B168**, 556, (1986).

[9] *Nikolaev V.A., Tkachev O.G.*: JINR Preprint E4-89-848, Dubna, (1989).;
Nikolaev V.A., Tkachev O.G.: Sov.J.Part.Nucl. **21(6)**, 643, (1990).

[10] *Nikolaeva R.M., Nikolaev V.A., Tkachev O.G.*: Sov.J.Part.Nucl. **23(2)**, 239, (1992).